Contribution to Uncertainty Analysis

Possolo, Antonio to: Bill.Lehr@noaa.gov, James J Riley , savas@newton.berkeley.edu, Alberto Aliseda, Juan Lasheras, Poojitha Yapa, Wereley@purdue.edu, Espina, Pedro I. , Franklin Shaffer, Paul Bommer, ira leifer , Mark K Sogge, Marcia K McNutt, Chris Barker

06/07/2010 02:49 PM

Dear Colleagues,

With apologies for my belated attempt to catch up with all of you, I attach some comments on uncertainty analysis where some of the issues and needs may be worth touching upon during the telecon that will start in about 15 minutes.

All standard disclaimers apply, including that I speak only for myself, not for NIST as a whole.

- Antonio

- Antonio Possolo, PhD -- Chief
Statistical Engineering Division
Information Technology Laboratory
National Institute of Standards & Technology
Telephone: 301-975-2853

UncertaintyAnalysis-Possolo2010Jun07.pdf

1 Scope

Review of Monte Carlo method to evaluate uncertainty of PIV flow measurements that were described in the May 26th, 2010, draft report entitled *Estimated Leak Rates and Lost Oil from the Deepwater Horizon Spill*, and discussion of outstanding issues.

2 Approach

The PIV estimate of oil flowing from the fissure near the lettering on the central portion of the drilling riser bent above the BOP, is $Q = VAX$, where V and A denote the average velocity and cross-sectional area of of the corresponding jet, and X denotes the volume fraction of oil in the jet.

The uncertainty analysis presented in that report was based on an established, internationally accepted approach to uncertainty propagation [Joint Committee for Guides in Metrology, 2008a] that is based on a linear approximation to Q as a function of all the participating quantities.

An alternative approach that is based on statistical modeling of the contributions from the recognized sources of uncertainty, and on Monte Carlo sampling [Joint Committee for Guides in Metrology, 2008b] may prove advantageous over the foregoing, approximate analysis, because it allows greater freedom in modeling the different sources of uncertainty, and involves no linearization of the relevant relations. In addition, this alternative approach generally is much easier to explain than the analytical approach, to decision makers, the media, the public, and the courts.

This alternative approach amounts to modeling V, A, and X (and any other quantities that these may, in turn, depend on), as random variables, and assigning suitable probability distributions to them, typically such that their mean values are their measured values, and their standard deviations are their respective measurement standard uncertainties. In general, any correlations that may exist between them also must be modeled [Possolo, 2010]. Then, samples are drawn from these distributions, and the corresponding values of Q obtained, whose standard deviation then is the measurement uncertainty associated with the estimate of Q.

3 Results

In this case, the two approaches produce similar results: the original, analytical approach yielded **41 %** as the relative standard uncertainty associated with the estimate of Q, while the statistical modeling, Monte Carlo sampling approach yielded **42 %**. In general, they may differ markedly — and when they do, the latter generally is preferable to the former.

4 Issues

1. Boundary Layer Bias. The PIV measurements of velocity are based on transits of features that are visible to the camera, hence that lie in the vicinity of the interface between the jet and the ocean water. For this, and for the reasons, related to variations in the density ratio, which Ömer Savaş mentioned in his *2010 Gulf Mexico Oil Spill Estimate* from earlier today, the use of uncorrected PIV velocities will lead to an underestimate of the amount of outflowing oil. Even if such correction proves impracticable, this should be expressed in the uncertainty assessment.

2. Cross-Sectional Area. The fissure that was the focus of the analysis under discussion here is markedly elongated. Based on examination of a single image that shows it only partially, may be closer to an ellipse (whose axial ratio may be about 3) than to a circle. If the cross-sectional shape of the jet, in the vicinity of where the PIV measurements have been made, should have been elliptical rather than circular, then this, too will bias the estimate of the flow, this time upwards — the biasing multiplicative factor (which is 1 when the cross-section is circular) depends only on the ellipse's axial ratio. If there is doubt about the shape of the cross-section, then this, too, should be incorporated in the uncertainty statement.

3. Frame Rates. If the uncertainty concerning frame rates is modeled by a uniform (or, rectangular) distribution with endpoints 14 and 48 frames per second, then the relative uncertainty of Q becomes **54 %**.

4. View Angle Corrections. Two cosine corrections are involved in the reductions of the PIV data: one concerns the view angle of the approximately linear transits of the features that PIV tracks; the other concerns the view angle of the apparent diameter of the jet. Since these need not be identical, in my analysis I have modeled them as independent but not identical random variables, with standard deviation 5° each.

5. Dispersion of Velocity Values. There is no information in the draft report entitled *Estimated Leak Rates and Lost Oil from the Deepwater Horizon Spill*, of the dispersion of the individual estimates of velocity corresponding to the individual features that were tracked for the PIV analysis. The assumption has been made that the relative standard uncertainty of the transit length is 5 %, but it is unclear how this relates to that dispersion of values. It will also matter whether the tracked features were close in space and time of one another, or not.

6. Volume Fraction of Oil. The original analysis assumes that the volume fraction of oil in the jet is $25\,\% \pm 0.4 \times 0.25\,\%$. The independent analysis that Paul Bommer made of this quantity suggests 67 % (based on the PENCOR data) or 59.7 % (based on the Schlumberger data). The estimates that Juan Lasheras *et al.* produced earlier today for the open end of the riser also hover at around 60 %. If the volume fraction conceivably could be any of these, and each of these would have relative uncertainty of 40 %, then the relative uncertainty associated with the estimate of Q would become **68 %**.

5 Needs

- Assessing the uncertainty of the total amount of oil flowing through the multiple exit points that have been detected requires that correlations between the amounts exiting from different locations be estimated. The pulsating pattern that Juan Lasheras *et al.* have detected in the temporal evolution of the velocity and composition of the mixture exiting through the open end of the riser add cogency to this need.
- Since very different methods of estimation are being used (including PIV, and Ömer Savaş's turbulent jet self-similarity analysis), there is a need

to harmonize or reconcile these independent estimates, and to do so in a manner that still allows qualifying the result with a defensible statement of uncertainty — the method proposed by Lindley [1983] may come in handy for this purpose.

References

Joint Committee for Guides in Metrology. *Evaluation of measurement data — Guide to the expression of uncertainty in measurement*. International Bureau of Weights and Measures (BIPM), Sèvres, France, September 2008a. URL `http://www.bipm.org/en/publications/guides/gum.html`. BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, JCGM 100:2008, GUM 1995 with minor corrections.

Joint Committee for Guides in Metrology. *Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — Propagation of distributions using a Monte Carlo method*. International Bureau of Weights and Measures (BIPM), Sèvres, France, 2008b. URL `http://www.bipm.org/en/publications/guides/gum.html`. BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, JCGM 101:2008.

D. V. Lindley. Reconciliation of probability distributions. *Operations Research*, 31(5):866–880, September-October 1983.

A. Possolo. Copulas for uncertainty analysis. *Metrologia*, 47:262–271, 2010.